CERN-TH.7384/94

POLFIS-TH. 07/94

On the Geometry of Moduli Space of Vacua in N=2 Supersymmetric Yang–Mills Theory ⋆

A. Ceresole[1], R. D'Auria[1] and S. Ferrara[2]

[1] *Dipartimento di Fisica, Politecnico di Torino, Corso Duca Degli Abruzzi 24, 10129 Torino, Italy*
and
INFN, Sezione di Torino, Italy

[2] *CERN, 1211 Geneva 23, Switzerland*

Abstract

We consider generic properties of the moduli space of vacua in $N = 2$ supersymmetric Yang–Mills theory recently studied by Seiberg and Witten. We find, on general grounds, Picard–Fuchs type of differential equations expressing the existence of a flat holomorphic connection, which for one parameter (*i.e.* for gauge group $G = SU(2)$), are second order equations. In the case of coupling to gravity (as in string theory), where also "gravitational" electric and magnetic monopoles are present, the electric–magnetic S duality, due to quantum corrections, does not seem any longer to be related to $Sl(2, \mathbb{Z})$ as for $N = 4$ supersymmetric theory.

⋆ Supported in part by DOE grants DE-AC0381-ER50050 and DOE-AT03-88ER40384,Task E.

August 1994

Recently it has been shown that general properties of electric-magnetic duality, which is eventually linked to a conjectured dilaton-axion duality in superstring theories[1], can be described in a fairly general way in $N = 4$[2] and $N = 2$[3] supersymmetric Yang-Mills theories. However, $N = 2$ Yang-Mills theories look much more interesting since both perturbative and non perturbative phenomena, absent for $N = 4$[4] , play an important rôle in the discussion and determination of the electric– magnetic duality. Due to the fact that the moduli space of $N = 2$ Yang–Mills vacua is given by an $N = 2$ Kählerian space of a particular kind[5] , it turns out that electric–magnetic duality is described by a monodromy group Γ which is a subgroup of $Sp(2r, \mathbb{Z})$ where r is the rank of the Yang–Mills group G. For the case $r = 1$ ($G = SU(2)$), Seiberg and Witten identified Γ to be the group $\Gamma_2 \subset Sp(2, \mathbb{Z}) \simeq Sl(2, \mathbb{Z})$[3]. The monodromies are reminiscent of a similar problem that arises in the analysis of Calabi–Yau moduli space[6] where the monodromy group Γ, related to the target space duality group[7] , is a discrete subgroup of $Sp(2h_{21} + 2, \mathbb{Z})$ and is related to the three–form cohomology.

In this paper we show that, as expected, the monodromy related to electric–magnetic duality arises from "Picard–Fuchs" equations[8] which are associated to the rigid special geometry of $N = 2$ supersymmetric Yang–Mills theories. Following lines similar to those concerning the special geometry of Calabi–Yau moduli space[5], we shall first give a resumé of "rigid special geometry" in a coordinate free way and then write the associated system of differential equations, which always can be interpreted as the existence of a flat holomorphic connection on a certain holomorphic bundle.

Let us first remind that if one has n abelian vector multiplets (here $n = r$ since we consider generic flat directions of a pure Yang–Mills theory with gauge group G broken to $U(1)^r$) their scalar fields, in a supergravity basis, describe a Kählerian sigma model

$$G_{A\overline{B}} \partial X^A \partial \overline{X}^B \tag{1}$$

with metric

$$G_{A\overline{B}} = -2 \text{ Im } \partial_A \partial_{\overline{B}} F = \partial_A \partial_{\overline{B}} i (F_C \overline{X}^C - \overline{F}_C X^C) \tag{2}$$

where F is a holomorphic function of X ($\overline{\partial} F = 0$). These coordinates are the analogue of the "special coordinates" in the context of Calabi–Yau moduli space. A general coordinate free way of describing the special geometry for that case was given in [5] and the associated system of Picard–Fuchs equations, together with the

flat holomorphic geometry, was discussed in [5,8] . The Riemann tensor of special geometry satisfies

$$R_{\alpha\overline{\beta}\gamma\overline{\delta}} = g_{\alpha\overline{\beta}}g_{\gamma\overline{\delta}} + g_{\gamma\overline{\beta}}g_{\alpha\overline{\delta}} - e^{2K}W_{\alpha\beta\epsilon}W_{\overline{\beta\delta\epsilon}}G^{\epsilon\overline{\epsilon}} \ , \tag{3}$$

where $G_{\epsilon\overline{\epsilon}} = \partial_\epsilon\partial_{\overline{\epsilon}}K$ is the Kähler metric and $W_{\alpha\beta\gamma}$ is a totally symmetric holomorphic tensor.

Here we consider rigid special geometry, where the moduli space is simply a Kähler rather than a Kähler -Hodge manifold, and the constraint (3) becomes

$$R_{\alpha\overline{\beta}\gamma\overline{\delta}} = -W_{\alpha\beta\epsilon}W_{\overline{\beta\delta\epsilon}}G^{\epsilon\overline{\epsilon}} \ . \tag{4}$$

In the X coordinates $G_{\epsilon\overline{\epsilon}}$ is given by eq. (2) and

$$W_{ABC} = \partial_A\partial_B\partial_C F \ . \tag{5}$$

To promote formulae (2), (5) to arbitrary coordinates, one introduces n holomorphic functions $X^A(z)$ and a function $F(X^A(z))$. Then the Kähler potential is

$$K(z,\overline{z}) = i(F_A\overline{X}^A - \overline{F}_A X^A) \quad (F_A = \frac{\partial F}{\partial X^A}) \ , \tag{6}$$

and

$$\begin{aligned} G_{\alpha\overline{\beta}} &= \partial_\alpha X^A \partial_{\overline{\beta}}\overline{X}^B \frac{\partial}{\partial X^A}\frac{\partial}{\partial \overline{X}^B}K \\ W_{\alpha\beta\gamma} &= \partial_\alpha X^A\partial_\beta X^B\partial_\gamma X^C\partial_A\partial_B\partial_C F \end{aligned} \ . \tag{7}$$

It is convenient, as in ref. [5,8], to introduce the flat vielbein

$$e_\alpha^A = \partial_\alpha X^A \tag{8}$$

which is a $n\times n$ matrix. The Christoffel connection $\Gamma^\gamma_{\alpha\beta}$ whose Riemann tensor satisfies

$$\partial_{\overline{\gamma}}\Gamma^\delta_{\alpha\beta} = G^{\delta\overline{\delta}}R_{\alpha\overline{\gamma}\beta\overline{\delta}} = -W_{\alpha\beta\epsilon}\overline{W}_{\overline{\gamma}\overline{\delta}\overline{\epsilon}}G^{\epsilon\overline{\epsilon}}G^{\delta\overline{\delta}} \tag{9}$$

can be written as

$$\Gamma^\delta_{\alpha\beta}(z,\overline{z}) = T^\delta_{\alpha\beta}(z,\overline{z}) + \widehat{\Gamma}^\delta_{\alpha\beta}(z) \ , \tag{10}$$

where

$$\begin{aligned} T^\delta_{\alpha\beta}(z,\overline{z}) &= e^A_\alpha e^B_\beta\partial_B G_{A\overline{D}}G^{-1\overline{D}C}e^{-1\delta}_{\ \ C} \\ \widehat{\Gamma}^\delta_{\alpha\beta}(z) &= \partial_\beta e^A_\alpha e^{-1\delta}_{\ \ A} \end{aligned} \ . \tag{11}$$

From (11) eq. (9) immediately follows. $\widehat{\Gamma}$ defines a flat connection in an $n \times n$ space: $R(\widehat{\Gamma}) = 0$.

If one introduces the $2n$ objects $X^A(z), F_A(z)$ and the $2n$ dimensional vector $V = (X^A, F_A)$, it is easy to show that the following identities hold

$$\begin{aligned} D_\alpha V &= V_\alpha \\ D_\alpha V_\beta &= -i\, W_{\alpha\beta\gamma} V^\gamma \qquad V^\gamma = G^{\gamma\overline{\gamma}}\, \overline{V}_{\overline{\gamma}} \\ D_\alpha \overline{V}_{\overline{\gamma}} &= 0 \end{aligned} \tag{12}$$

where D_α is the covariant derivative in the original Kähler manifold. To this non holomorphic system of identities it is associated an holomorphic system, which is obtained by replacing D_α with the flat covariant derivative $\widehat{D}_\alpha$ where $\Gamma \to \widehat{\Gamma}$, and V^γ is replaced by a holomorphic vector

$$V^\alpha = (0, e^{-1\alpha}_{\ A}) \; . \tag{13}$$

Since the first equation is left invariant by constant translations $V \to V + c$, it is actually possible to consider (V, V_α, V^α) as $(2N+1)$ vectors so that

$$\begin{aligned} V &= (1, X^A, F_A) \\ V_\alpha &= (0, e^A_\alpha, e^B_\alpha F_{AB}) \\ V^\alpha &= (0, 0, e^{-1\alpha}_{\ A}) \end{aligned} \tag{14}$$

In terms of the $(2n+1) \times (2n+1)$ matrix

$$\mathcal{V} = \begin{pmatrix} V \\ V_\beta \\ V^\beta \end{pmatrix} \; , \tag{15}$$

the holomorphic system can be written as

$$\mathcal{D}_\alpha \mathcal{V} = 0 \; , \tag{16}$$

with

$$\mathcal{D}_\alpha = \partial_\alpha - \mathcal{A}_\alpha \tag{17}$$

and the flat connection $\mathcal{A}_\alpha$ given by

$$\mathcal{A}_\alpha = \begin{pmatrix} 0 & \delta^\gamma_\alpha & 0 \\ 0 & \widehat{\Gamma}^\gamma_{\alpha\beta} & (W_\alpha)_{\beta\gamma} \\ 0 & 0 & -\widehat{\Gamma}^\beta_{\alpha\gamma} \end{pmatrix} \; . \tag{18}$$

Note that there is a $ISp(2n)$ acting on the right hand side of V (or $\mathcal{V}$) represented as $\begin{pmatrix} 1 & 0 \\ C & M \end{pmatrix}$ with $M \subset Sp(2n)$. This follows from the fact that the submatrix $\begin{pmatrix} \widehat{\Gamma}^{\gamma}_{\alpha\beta} & (W_\alpha)_{\beta\gamma} \\ 0 & -\widehat{\Gamma}^{\beta}_{\alpha\gamma} \end{pmatrix}$ is valued in the lie algebra of $Sp(2n)$ (with respect to the metric $Q = \begin{pmatrix} 0 & -1_n \\ 1_n & 0 \end{pmatrix}$).

In special coordinates $e^A_\alpha = \delta^A_\alpha$, $\widehat{\Gamma} = 0$ and the connection $\mathcal{A}_\alpha$ reduces to

$$\mathcal{A}_\alpha = \begin{pmatrix} 0 & \delta^\gamma_\alpha & 0 \\ 0 & 0 & W_{\alpha\beta\gamma} \\ 0 & 0 & 0 \end{pmatrix} \tag{19}$$

which is nilpotent of degree three ($(\mathcal{A}_\alpha)^3 = 0$).

Eq. (16) can be rewritten as a system of third order differential equations for the upper component of $\mathcal{V}$,

$$\widehat{D}_\alpha (W^{-1\widehat{\gamma}})^{\epsilon\beta} \widehat{D}_{\widehat{\gamma}} \partial_\beta V = 0 \ , \tag{20}$$

(where $\widehat{\gamma}$ is a priori not summed over). Actually, since the first equation in (12) can be used to just start with V_α, (20) can be reduced to a second order differential equation for $V_\beta = \partial_\beta V$. We can then delete the first entry in (14)

$$\begin{aligned} V_\alpha &= (e^A_\alpha, e^B_\alpha F_{AB}) \\ V^\alpha &= (0, e^{-1\alpha}_A) \end{aligned} \tag{21}$$

and write the connection as $\mathcal{A}_\alpha = \begin{pmatrix} \widehat{\Gamma}^{\gamma}_{\alpha\beta} & (W_\alpha)_{\beta\gamma} \\ 0 & -\widehat{\Gamma}^{\beta}_{\alpha\gamma} \end{pmatrix}$, which reduces in special coordinates to

$$\mathcal{A}_\alpha = \begin{pmatrix} 0 & (W_\alpha)_{\beta\gamma} \\ 0 & 0 \end{pmatrix} \tag{22}$$

which is then nilpotent of degree two, and $W_{\alpha\beta\gamma}$ is an n–dimensional abelian subalgebra. The physical meaning of $W_{\alpha\beta\gamma}$ is that they are related to the Riemann tensor over the moduli space by eq. (4).

In the case of one variable ($n = 1$), equation (20) becomes

$$(\widehat{D} W^{-1} \widehat{D} \widehat{\partial}) V = 0 \ , \tag{23}$$

and setting $U = \partial V$ it becomes

$$(\partial + \widehat{\Gamma})W^{-1}(\partial - \widehat{\Gamma})U = 0 \ . \tag{24}$$

This yields a second order equation

$$\partial^2 U + a_1 \partial U + a_0 U = 0 \ , \tag{25}$$

with

$$\begin{aligned} a_1 &= -\partial \log W \\ a_0 &= \partial \log W \widehat{\Gamma} - \partial \widehat{\Gamma} + \widehat{\Gamma}^2 \qquad \widehat{\Gamma} = \partial \log e \end{aligned} \tag{26}$$

so that knowing a_1, a_0 one can compute W and e (e=1 in special coordinates). Note that the general solution in the one parameter case is

$$U = \partial V = (e, e\frac{\partial^2 F}{\partial X^2}) \ , \tag{27}$$

where e is the vielbein component that here plays the rôle of a rescaling factor. Taking the ratio of the two solutions one gets that $\tau = \frac{\partial^2 F}{\partial X^2}$ is the uniformizing variable for which the differential equation reduces to $\frac{d^2}{d\tau^2}(\ ) = 0$. This is consistent with the fact that the metric of the effective supergravity theory is

$$G_{z\bar{z}} = |e(z)|^2 \mathrm{Im}\ \tau = |e(z)|^2 \mathrm{Im}\ \frac{\partial^2 F}{\partial X^2} > 0 \tag{28}$$

and therefore manifestly positive[3].

As an explicit example, let us derive the differential equation (25) for the particular one parameter case of Seiberg and Witten [3]. Consider the family E_u of genus one Riemann surfaces

$$y^2 = (x+1)(x-1)(x-u) \ , \tag{29}$$

which, in homogeneous coordinates $(x \rightarrow \frac{x}{z}, y \rightarrow \frac{y}{z})$ can be described by the vanishing of the homogeneous polynomial $\mathcal{W}$ in CP^2,

$$\mathcal{W}(x,y,z) = -zy^2 + x(x^2 - z^2) - u\ z(x^2 - z^2) \ . \tag{30}$$

For convenience, we change variables to $(x \rightarrow x+z, z \rightarrow x-z)$, obtaining

$$\mathcal{W} = -(x-z)y^2 + xz(x-z) - u\ xz(x-z) \ . \tag{31}$$

The differential equation associated to (31) can now be derived using standard techniques, familiar from topological Landau–Ginzburg theories [9]. Define the integrals

$$U_0 = \int \frac{\omega}{\mathcal{W}} \quad , \quad U_1 \equiv \frac{dU_0}{du} = \int \frac{\omega}{\mathcal{W}^2} xz(x-z) \tag{32}$$

where ω is a volume form, which form a basis of the cohomology $H^1(E_u)$. By differentiating under the integral sign and using the "vanishing relations" $\frac{\partial \mathcal{W}}{\partial x} = \frac{\partial \mathcal{W}}{\partial y} = \frac{\partial \mathcal{W}}{\partial z} = 0$, one can show that the vector $\begin{pmatrix} U_0 \\ U_1 \end{pmatrix}$ satisfies a regular, singular matrix differential equation

$$\frac{d}{du}\begin{pmatrix} U_0 \\ U_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{2u}{(1-u^2)} & \frac{1}{4(1-u^2)} \end{pmatrix} \begin{pmatrix} U_0 \\ U_1 \end{pmatrix} \ , \tag{33}$$

which is equivalent to the second order differential equation

$$\frac{d^2 U_0}{du^2} - \frac{2u}{(1-u^2)} \frac{dU_0}{du} - \frac{1}{4(1-u^2)} U_0 = 0 \ . \tag{34}$$

Comparing (34)with (25), (26) one can read out that

$$W = \frac{1}{(u^2-1)} \tag{35}$$

and that $\widehat{\Gamma}$ satisfies

$$-\frac{2u}{u^2-1}\widehat{\Gamma} - \partial\widehat{\Gamma} + \widehat{\Gamma}^2 = \frac{1}{4(u^2-1)} \ . \tag{36}$$

Writing $\widehat{\Gamma} = \partial log\ e$, then (36) coincides with equation (34) for U_0, in agreement with the fact that, according to (27), e is one of the solutions of (34).

As a check, we may compute the asymptotic behaviour of the solutions $(U_0^{(1)}, U_0^{(2)})$ of this fuchsian equation around the singular points $u = 1, -1, \infty$. We find

$$\begin{array}{rl} u \to \pm 1 & (U_0^{(1)}, U_0^{(2)}) \approx (c_1, log(u \mp 1)) \\ u \to \infty & (U_0^{(1)}, U_0^{(2)}) \approx (u^{-1/2}, u^{-1/2} logu) \end{array} \ . \tag{37}$$

Recalling that $U_0^{(1)} = \partial V^{(1)}, U_0^{(2)} = \partial V^{(2)}$ and the third period $V^{(3)} = c$, one finds

$$\begin{array}{rl} u \to \pm 1 & (V^{(1)}, V^{(2)} + V^{(3)}) \approx (u \mp 1, c + c'(u \mp 1) log(u \mp 1)) \\ u \to \infty & (V^{(1)}, V^{(2)} + V^{(3)}) \approx (u^{1/2}, u^{1/2} logu) \end{array} \tag{38}$$

in agreement with the behaviour of the periods (a, a_D) of [3].

The change of variables $u \to 1-2z$, puts eq.(34) into the form of an hypergeometric equation of parameters $(\frac{1}{2},\frac{1}{2},1)$ so that $U_0^1 = \ _2F_1\left[\frac{1}{2},\frac{1}{2},1;\frac{1-u}{2}\right]$. Using the standard relations among hypergeometric functions [10] one could also reconstruct the monodromy matrices of the periods in a symplectic basis as given in [3].

The geometry of the moduli space is actually remarkably different when gravitational degrees of freedom are introduced [2]. The reason is that in that case there are always two additional $U(1)$ factors, one coming from $G_{\mu i}$ and the other from $B_{\mu i}$. One is the $N=2$ graviphoton and the other is the vector partner of the dilaton–axion multiplet. Therefore one gets a $U(1)^{r+2}$ abelian algebra and at least $r+1$ vector multiplets. In string theory with maximal G, $r=22$ and the special Kähler manifold has dimension $r+1$. If the gauge group would be taken to be $SU(2)$ $(r=1)$, then the holomorphic prepotential would be of the form[3,4] (in special coordinates $s=\frac{X_1}{X_0}, t=\frac{X_2}{X_0}$, and for instanton number n)

$$F(s,t)=st^2+f_{one\ loop}(t)+\sum_{n=1}^{\infty}C_n\ t^2(\frac{\Lambda^2}{t^2})^{2n}e^{2\pi ins} \tag{39}$$

with $s=i\frac{4\pi}{g^2}+\frac{\theta}{2\pi}$, where $f_{one\ loop}(t)$ does not violate the s (dilaton–axion) Peccei–Quinn symmetry and the non perturbative part gives the space time instanton contribution. The generalization of this formula for $G=SU(N)$ is straightforward[4]. Moreover, the metric of the moduli space will be that of special geometry [6,8] .

Unlike the rigid case discussed in [3], it is natural to conjecture here a monodromy in two variables and a central charge of the type [3,11]

$$Z=\sum_{A=1}^{3}N^A_{(m)}F_A-M^A_{(e)}X_A \tag{40}$$

where $A=0,s,t$. The new $0,s$ components correspond to gravitationally electrically and magnetically charged states. This formula is analogous to the one suggested in [11] for the massive Kaluza-Klein and winding states for $(2,2)$ supersymmetric compactifications. There, (X^A,F_A) play the rôle of periods of holomorphic three-form and duality is manifest with respect to monodromy in the moduli space of $(2,2)$ vacua[6,8].

In this case the monodromy group, *i.e.* the duality group, would not be in $Sp(2,\mathbb{Z})$ but rather in $Sp(6,\mathbb{Z})$ with a Picard–Fuchs system identical in form to the two– parameter case of a Calabi–Yau moduli space. In this case there is an intriguing

analogy between the moduli space of $N=2$ supersymmetric Yang–Mills theory coupled to supergravity (with gauge group G of rank r) and Calabi–Yau moduli space for the three-form cohomology with hodge number $h_{21}=r+1$. The modular forms with respect to Γ should reconstruct the full holomorphic function $F(s,t)$. We further remark that the $Sl(2,\mathbb{Z})$ symmetry associated to dilaton–axion (S) duality is peculiar of $N=4$ theories only, because of the absence of quantum corrections. Indeed, for $N=2$ supersymmetric theories, the monodromy group associated to the periods $(1,s,t,F_s,F_t,2F-sF_s-tF_t)$ is expected to be a discrete group $\Gamma\in Sp(6,\mathbb{Z})$, which will not in any way be related to $Sl(2,\mathbb{Z})$ or any of its subgroups. It is merely a property of the tree-level uncorrected prepotential $F(s,t)=st^2$ to exhibit a non– linear $Sl(2,\mathrm{R})$ symmetry (containing $Sl(2,\mathbb{Z})$) previously found in $N=4$ supergravity[12]. This is similar to the example of the one–parameter Calabi–Yau moduli space whose metric for large volume approaches the metric of $\frac{SU(1,1)}{U(1)}$ homogeneous space[13].

Acknowledgements

We would like to thank R. Stora for useful discussions.

References

[1] A. Font, L. Ibanez, D. Lust, F. Quevedo, Phys. Lett. 249*B* (1990) 35.

[2] A. Sen, Nucl. Phys. *B*388 (1992) 457 and Phys. Lett. 303*B* (1993) 22; A. Sen and J. H. Schwarz, Nucl. Phys. *B*411 (1994) 35; Phys. Lett. 312*B* (1993) 105; M. J. Duff and R. R. Khuri, Nucl. Phys. *B*411 (1994) 473; J. Gauntlett and J. A. Harvey, preprint EFI-94-36; L. Girardello, A. Giveon. M. Porrati and A. Zaffaroni, preprint NYU-TH-94/06/02.

[3] N. Seiberg and E. Witten, *Electric–Magnetic Duality, Monopole Condensation, And Confinement In* $N = 2$ *Supersymmetric Yang–Mills Theory* , preprint RU-94-52, IAS-94-43 , hep-th 9407087.

[4] N. Seiberg, Phys. Lett. 206*B* (1988) 75.

[5] B. de Wit and A. van Proeyen, Nucl. Phys. *B*245 (1984) 89; E. Cremmer, C. Kounnas, A. van Proeyen, J. Derendinger, S. Ferrara, B. De Wit and L. Girardello, Nucl. Phys. *B*250 (1985) 385; B. de Wit, P. G. Lauers and A. Van Proeyen, Nucl. Phys. *B*255 (1985) 569; S. Cecotti, S. Ferrara and L. Girardello, Int. J. Mod. Phys. *A*4 (1989) 2475.

[6] N. Seiberg, Nucl. Phys. *B*303 (1988) 206; A. Strominger, Comm. Math. Phys. 133 (1990) 163; L. Castellani, R. D'Auria and S. Ferrara, Phys. Lett. 241*B* (1990) 57; Class. Quantum Grav. 1 (1990) 317; P. Candelas and X. de la Ossa, Nucl. Phys. *B*355 (1991) 455; S. Ferrara and J. Louis, Phys. Lett. 278*B* (1992) 240; A. Ceresole, R. D'Auria, S. Ferrara, W. Lerche and J. Louis, Int. J. Mod. Phys. *A*8 (1993) 79

[7] For a review on target space duality see A. Giveon, M. Porrati and E. Rabinovici, preprint RI-I-94, hep-th/9401139.

[8] For a recent review, see A. Ceresole, R. D'Auria, S. Ferrara, W. Lerche, J. Louis and T. Regge, *Picard–Fuchs Equations, Special Geometry and Target Space Duality* , preprint CERN-TH.7055/93, POLFIS-TH.09/93, to appear on *Essays on Mirror Manifolds* , vol. II, S. T. Yau editor, International Press (1994).

[9] A. C. Cadavid and S. Ferrara, Phys. Lett. 267*B* (1991) 193; W. Lerche, D. Smit and N. Warner, Nucl. Phys. *B*372 (1992) 87.

[10] A. Erdélyi, F. Obershettinger, W. Magnus and F. G. Tricomi, *Higher Trascendental Functions*, , McGraw Hill, New York, (1953).

[11] S. Ferrara, C. Kounnas, D. Lust and F. Zwirner, Nucl. Phys. $\underline{B365}$ (1991) 431 .

[12] E. Cremmer, J. Scherk and S. Ferrara, Phys. Lett. $\underline{074B}$ (1978) 61.

[13] P. Candelas, X. de la Ossa, P. S. Green and L. Parkes, Phys. Lett. $\underline{258B}$ (1991) 118; Nucl. Phys. $\underline{B359}$ (1991) 21.